

# Collective excitations in the inner crust of neutron stars : supergiant resonances

E. Khan<sup>a</sup>, N. Sandulescu<sup>a,b,c</sup> and Nguyen Van Giai<sup>a</sup>

*a) Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, 91406 Orsay Cedex,  
France*

*b) Institute of Physics and Nuclear Engineering, 76900 Bucharest, Romania*

*c) Department of Physics, University of Washington, Seattle, WA 98195, USA*

## Abstract

We investigate the nuclear collective excitations of Wigner-Seitz cells containing nuclear clusters immersed in a gas of neutrons. This baryonic non-uniform system is specific to the structure of inner crust matter of neutron stars. The collective excitations are studied in the framework of a spherical Hartree-Fock-Bogoliubov + Quasiparticle Random Phase Approximation, formulated in coordinate representation. The calculations are done for two representative Wigner-Seitz cells with baryonic density equal to  $0.02 \text{ fm}^{-3}$  and  $0.08 \text{ fm}^{-3}$ . It is shown that the excitations with low multipolarities are concentrated almost entirely in one strongly collective mode which exhausts a very large fraction of the energy-weighted sum rule. Since these collective modes are located at very low energies compared to the giant resonances in standard nuclei, they may affect significantly the specific heat of baryonic inner crust matter of neutron stars.

*PACS numbers:* 21.60Jz, 26.60+c, 24.30Cz, 21.10Re

The cooling of low-mass neutron stars is influenced strongly by the superfluid properties of inner crust matter [1]. The superfluid properties and their influence on the specific heat were analyzed rather intensively in the past years by using various frameworks, e.g. semi-classical pairing models [2,3], Bogoliubov-type calculations based on a Woods-Saxon mean field [4,5], and the self-consistent Hartree-Fock-Bogoliubov (HFB) approach [6,7]. These calculations showed that the specific heat of baryonic inner crust matter can be reduced by orders of magnitudes due to pairing correlations.

In all the calculations mentioned above the specific heat was evaluated by considering only non-interacting quasiparticles states. However, the specific heat can be also strongly affected by the collective modes created by the residual interaction between the quasiparticles, especially if these modes appear at low-excitation energies. To our knowledge, at present there is not any microscopic study of such nuclear collective modes induced in the inner crust matter of neutron stars. The scope of this Letter is to present such an investigation. The specific heat of the inner crust is also largely determined by the electrons motion and, to a lesser extent, by the lattice vibrations [1,5,8,9]. These degrees of freedom of the inner crust matter are not discussed here.

In microscopic calculations the inner crust matter is usually treated in the Wigner-Seitz (WS) approximation [8,10]. Accordingly the inner crust is modelised by non-interacting cells containing a neutron-rich nucleus immersed in a dilute gas of neutrons and relativistic electrons. For baryonic densities ranging from  $1.4 \cdot 10^{-3} \rho_0$  to about  $0.5 \rho_0$ , where  $\rho_0 = 0.16 \text{ fm}^{-3}$  is the nuclear matter saturation density, the nuclear clusters are considered spherical [10,11]. At higher densities, the inner crust matter can develop various non-spherical phases (e.g., rods, slabs, tubes, bubbles) [8]. In this Letter we will focus on the nuclear collective modes developed in WS cells containing spherical nuclear clusters. For illustration we will consider two representative WS cells chosen from Ref. [10], i.e., one formed by 50 protons and 1750 neutrons and an other formed by 32 protons and 950 neutrons. These two cells will be quoted below as  $^{1800}\text{Sn}$  and  $^{982}\text{Ge}$ . The density of the neutron gas far from the nuclear cluster is of about  $0.018 \text{ fm}^{-3}$  in the first cell and about  $0.074 \text{ fm}^{-3}$  in the second. The radius of the first (second) cell is 27.6 (14.4) fm.

The collective response is calculated in the HFB+Quasiparticle Random Phase Approximation (QRPA) approach formulated in the coordinate representation [12]. In the first step of the calculation we solve the HFB equations for the ground state of the given Wigner-Seitz cell, considered as an isolated system. The HFB calculations are performed in coordinate representation and imposing Dirichlet-Neuman boundary conditions at the edge of the cell [10]. The effective forces used in the HFB calculations are the same as in Ref. [7], i.e., the Skyrme interaction SLy4 [13] in the particle-hole channel and a density-dependent contact force for the pairing interaction. The parameters of the pairing interaction are chosen so as to obtain in neutron matter approximatively the same pairing gap as given by the Gogny force [6,14,15]. The HFB results for the pairing fields and particle densities of the two WS cells mentioned above are shown in Figures 1 and 2.

The solutions of the HFB equations are used in the next step to construct the QRPA Green function. In the Green function are introduced all the quasiparticle states with energies up to 60 MeV, which is the cut-off energy employed in the HFB calculations. Both HFB and QRPA equations are solved on a radial mesh with a 0.2 fm step and the Green function is calculated by using an energy smoothing factor equal to 150 keV. The residual

interaction between the quasiparticles is calculated from the second derivative of the HFB energy functional with respect to the particle and pairing densities. For calculating the residual interaction in the particle-hole channel we apply the Landau-Migdal approximation [16], which simplifies the numerical treatment of the velocity-dependent terms of the Skyrme interaction. Due to this approximation the full self-consistency of HFB+QRPA calculations is slightly broken. As a result, the spurious mode associated to the translational invariance is not located exactly at zero energy. In order to bring it to zero we renormalise the particle-hole part of the residual interaction by a factor of 0.93. The Coulomb and the spin-orbit residual interactions between quasiparticles are dropped since they play a minor role compared to the other interactions. However, both the Coulomb and spin-orbit interactions are included in the HFB mean-field calculations. Further details on how the collective response is calculated in the HFB+QRPA approach can be found in Refs. [12,17].

In order to analyse the microscopic content of the response function we need to express the forward (X) and backward (Y) amplitudes of the QRPA operators [18] in coordinate representation. Thus it can be shown that the forward amplitudes corresponding to an exciting state  $\nu$  have the following form:

$$X_{ij}^\nu = \frac{1}{2} \int d\mathbf{r} \sum_\sigma \rho^\nu(\mathbf{r}) \left( V_j^*(\mathbf{r}\sigma) U_i^*(\mathbf{r}\sigma) - V_i^*(\mathbf{r}\sigma) U_j^*(\mathbf{r}\sigma) \right) + \kappa^\nu(\mathbf{r}) U_j^*(\mathbf{r}\bar{\sigma}) U_i^*(\mathbf{r}\sigma) + \bar{\kappa}^\nu(\mathbf{r}) V_j^*(\mathbf{r}\sigma) V_i^*(\mathbf{r}\bar{\sigma}) \quad (1)$$

where  $\rho^\nu$  is the particle-hole transition density for the state  $\nu$  while  $\kappa^\nu$  and  $\bar{\kappa}^\nu$  are the pair addition and pair removal transition densities [17]. The quantities  $U_i(r\sigma)$  and  $V_i(r\sigma)$  are the two components of the HFB wave functions and  $\sigma$  is the spin index. The time reversed wave functions are denoted by the spin index  $\bar{\sigma}$ .

Within the HFB+QRPA formalism presented above we first calculate the quadrupole neutron response in the cell  $^{1800}\text{Sn}$ . The corresponding transition operator is  $r^2 Y_{20}$ . The unperturbed HFB response, built by non-interacting quasiparticle states, and the QRPA response are shown in Figure 3. As can be clearly seen, when the residual interaction is introduced among the quasiparticles the unperturbed spectrum, distributed over a large energy region, is gathered almost entirely in a the peak located at about 3 MeV. This peak collects more than 99% of the total quadrupole strength and it exhausts about 70% of the energy-weighted sum rule. This mode is extremely collective. Thus, there are more than one hundred two-quasiparticle configurations which contribute to this mode, with the two main configurations contributing no more than 5%. Another indication of the extreme collectivity of this low-energy mode can be seen from its reduced transition probability,  $B(E2)$ , which is equal to  $25 \cdot 10^3$  Weisskopf units. This value of  $B(E2)$  is two orders of magnitude higher than in standard nuclei. We notice also that an extrapolation of the energy position based on the GDR systematics in finite nuclei, i.e.,  $65 \cdot A^{-1/3}$  MeV [19], would predict the low-energy peak at about 5 MeV. This underlines the fact that this WS cell cannot be simply considered as a giant nucleus. As discussed below, the reason is that in this cell the collective dynamics of the neutron gas dominates over the cluster contribution. Apart from the quadrupole mode discussed above, we have also investigated the response of the WS cell to the dipole and monopole excitations: they display similar features, leading to the same qualitative conclusions.

It is interesting to examine what would be the energy of the Bogoliubov-Anderson (BA) excitation mode [20,21] which the neutron superfluid gas would develop in the WS cell. For

a low-momentum  $p$  the energy of the BA mode is given approximatively by  $\omega = v_F p / \sqrt{3}$ , where  $v_F$  is the Fermi velocity. To estimate the BA mode in a WS cell one can use for  $p$  the values provided by the condition  $j_\lambda(kR)=0$ , where  $j_\lambda$  is the spherical Bessel function,  $\lambda$  is the multipolarity of the excitation,  $R$  is the radius of the cell, and  $k = p/\hbar$ . The value of  $v_F$  is obtained from the density of the neutron gas, taking the mass of neutrons equal to the bare mass. Using these approximations one gets for the lowest quadrupole mode an energy equal to 3.1 MeV, which is close to the energy extracted from the HFB+QRPA calculations. Thus the low-lying quadrupole excitations in the cell  $^{1800}\text{Sn}$  can be viewed as hydrodynamic sound-type modes produced by the neutron superfluid. The hydrodynamic picture is supported by the small coherence length of neutron gas superfluid, of about 3 fm, compared to the dimension of the WS cell. The value of the coherence length quoted above corresponds to the approximation  $\xi = \hbar v_F / (\pi \Delta)$  [22], where  $\Delta$  is the averaged pairing gap in the neutron gas region.

In order to study how the collective excitations would behave at higher baryonic densities of inner crust matter, we have also calculated the response for the cell  $^{982}\text{Ge}$ . As seen in Figs.1-2, in this cell the cluster and the neutron gas are less distinct than in the cell  $^{1800}\text{Sn}$ . Therefore one expects that the collective modes developed in the cell  $^{982}\text{Ge}$  are rather different from the hydrodynamic modes of the neutron gas superfluid.

The neutron quadrupole response of  $^{982}\text{Ge}$  provided by the HFB+QRPA calculations is shown in Figure 4. One of most striking feature seen in Figure 4 is the huge difference between the unperturbed and QRPA response. Thus one can see that by introducing the residual interaction between quasiparticle states, the unperturbed strength, dominated by the peak at 11 MeV, is shifted down and distributed through an energy region of about 9 MeV. The lowest quadrupole mode appears at 1.8 MeV. This mode is built in proportion of 23% by the two-quasiparticle configurations  $(15_{31/2} 17_{35/2})$  and  $(15_{29/2} 17_{33/2})$ . In these configurations, which form also the main part of the unperturbed spectrum, the single-particle states corresponding to the first (second) quasiparticle states are almost entirely occupied (empty). Due to the large degeneracy of these states, the residual interaction forms many particle-hole configurations which add coherently to the low-energy quadrupole mode. As expected, due to the dominance of these particle-hole configurations, the main features of the low-lying peak will be present also in a HF+RPA calculation. This is indeed the case, as seen in the upper part of Figure 4. It should be however noted that the pairing residual interaction significantly spreads the strength of the RPA peak located above 8 MeV.

In conclusion, the collective response of two representative WS cells has been calculated fully microscopically, using the HFB + QRPA approach. We found that in both Wigner-Seitz cells the residual interaction between the quasiparticle states generates a sort of supergiant resonances located at low energies. These energies have the same order of magnitude than the average pairing gap of neutron superfluid. Consequently, these collective modes can affect significantly the entropy and the specific heat of baryonic inner crust matter. A quantitative estimation of these effects would require finite-temperature HFB+QRPA calculations [23]. These calculations will be the subject of a future study.

One of us (NS) acknowledges valuable discussions with A.Bulgac.

## REFERENCES

- [1] D. G. Yakovlev and C. J. Pethick, *Annu. Rev. Astron. Astrophys* **42** (2004)169
- [2] R. A. Broglia, F. De Blasio, G. Lazzari, M. Lazzari, and P. M. Pizzochero, *D50* 4781 (1994); F. De Blasio, G. Lazzari, P. M. Pizzochero and R. A. Broglia, *Phys. Rev.* **D53** 4226 (1996)
- [3] O. Elgaroy, L. Engvik, E. Osnes, F. V. De Blasio, M. Hjorth-Jensen, and G. Lazzari, *Phys. Rev.* **D54** 1848 (1996)
- [4] F. Barranco, R. A. Broglia, H. Esbensen, and E. Viguzzi, *Phys. Rev.* **C58** 1257 (1998) ; F. Barranco et al, *Phys. Lett.* **B390** 13 (1997)
- [5] P. M. Pizzochero, F. Barranco, E. Vigetti, and R. A. Broglia, *ApJ* **569** (2002) 381
- [6] N. Sandulescu, Nguyen Van Giai, and R. J. Liotta, *Phys. Rev.* **C69** 045802 (2004)
- [7] N. Sandulescu, *Phys. Rev.* **C70** 025801 (2004)
- [8] C. J. Pethick and D. G. Ravenhall, *Annu. Rev. Nucl. Part. Sci.* **45** 429 (1995)
- [9] P. Magierski, *Int. J. Mod. Phys.* **E13** (2004) 371
- [10] J.W. Negele and D. Vautherin, *Nucl. Phys.* **A207** (1973) 298
- [11] F. Douchin and P. Haensel, *Phys. Lett.* **B485** 107 (2000)
- [12] E. Khan, N. Sandulescu, M. Grasso, Nguyen Van Giai, *Phys. Rev* **C66** (2002) 024309.
- [13] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, *Nucl. Phys.* **A635** (1998) 231.
- [14] G. F. Bertsch and H. Esbensen, *Ann. Phys. (N.Y.)* **209** 327 (1991)
- [15] E. Garrido, P. Sarriguren, E. Moya de Guerra, and P. Schuck, *Phys. Rev.* **C60** 064312 (1999)
- [16] S. O. Bäckman, A. D. Jackson and J. Speth, *Phys. Lett.* **B56** (1975) 209.
- [17] E. Khan, N. Sandulescu, Nguyen Van Giai, M. Grasso, *Phys. Rev.* **C69** (2004) 014314
- [18] P. Ring, P. Schuck, *The nuclear many-body problem*, Springer-Verlag (1980).
- [19] A. van der Woude, *Prog. Part. Nucl. Phys.* **18** (1987) 217
- [20] N. N. Bogoliubov, V. V. Tolmachev, and D. N. Shirkov, *New Method in the Theory of Superconductivity* ( New York, 1959)
- [21] P. W. Anderson, *Phys. Rev.* **112** (1958) 1900
- [22] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Addison-Wesley Publish. Co., 1989)
- [23] E. Khan, Nguyen Van Giai, M. Grasso, *Nucl. Phys.* **A731** (2004) 311

## Figure captions

**Figure 1.** Particle densities calculated with HFB for the WS cells  $^{1800}\text{Sn}$  (solid line) and  $^{982}\text{Ge}$  (dashed line).

**Figure 2.** Pairing field calculated with HFB for the WS cells  $^{1800}\text{Sn}$  (solid line) and  $^{982}\text{Ge}$  (dashed line).

**Figure 3.** Quadrupole strength distribution of neutrons for the cell  $^{1800}\text{Sn}$ . The full curve represents the QRPA strength and the dashed line is the HFB unperturbed strength.

**Figure 4.** The same as in Figure 3 but for the cell  $^{982}\text{Ge}$ . The inset shows the response calculated in the HF+RPA approach.









